The Golden Pi White Paper

$$\hat{\pi} = \frac{4}{\sqrt{\varphi}} = 3.144605511\ldots$$

A Unified Construction of the Circle Constant Through the Golden Ratio, the Great Pyramid, and the Eye of Horus

The Alpha Secret Research Collective

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Abstract. This white paper consolidates the full body of research on the Golden Pi constant $\hat{\pi} = 4/\sqrt{\varphi} = 3.144605511\ldots$, a constructible, algebraic circle constant derived entirely from the golden ratio φ. We present its algebraic origin, the *instrumentum identity* that produces it in closed form, its encoding in the Great Pyramid of Giza through the Royal Cubit and the Seked slope, the $abc = 64$ golden triangle that reproduces the pyramid face angle, and its relation to the Eye of Horus fractions. Every numerical claim is exact or explicitly bounded.

1. Introduction

The ratio of a circle's circumference to its diameter is conventionally taken to be $\pi \approx 3.14159265$, a *transcendental* number whose decimal expansion is neither terminating nor eventually periodic. Since Lindemann's 1882 proof that π is transcendental, it is known that π cannot be constructed with a compass and straightedge, and the classical problem of *squaring the circle* is thereby impossible in the Euclidean plane.

This paper examines an alternative circle constant derived from the golden ratio:

$$\boxed{\hat{\pi} \;=\; \frac{4}{\sqrt{\varphi}} \;=\; 3.144605511029693\ldots} \tag{1}$$

where $\varphi = (1+\sqrt{5})/2 \approx 1.61803398875$ is the golden ratio. Unlike the conventional π, the constant $\hat{\pi}$ is *algebraic*: it is built from a single square root of the golden ratio and is therefore constructible. Its deviation from the transcendental π is small:

$$\frac{\hat{\pi} - \pi}{\pi} = 9.59 \times 10^{-4} \quad (\approx 0.096\%). \tag{2}$$

This paper does not claim that $\hat{\pi}$ replaces the standard π in analytic mathematics. Rather, it documents the observation that the number $4/\sqrt{\varphi}$ arises with striking regularity across independent geometric, architectural, and symbolic systems, and that a family of closed-form identities reproduces it exactly.

2. The Golden Ratio and Its Algebraic Field

The golden ratio is the unique positive solution of $\varphi^2 = \varphi + 1$:

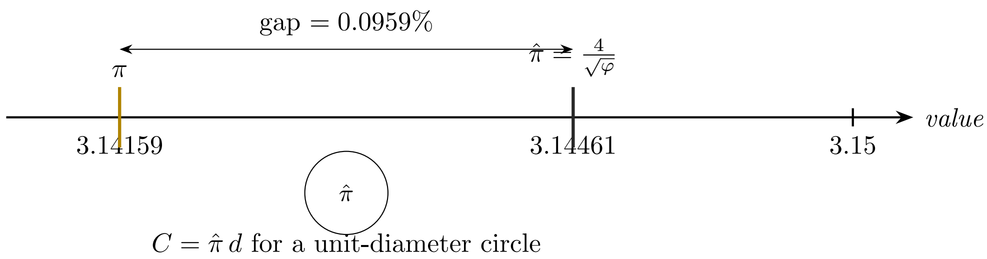


Figure 1: The two circle constants on a shared number line. The transcendental $\pi \approx 3.14159265$ and the algebraic Golden Pi $\hat{\pi} = 4/\sqrt{\varphi} \approx 3.14460551$ differ by a relative excess of 0.0959% (the factor $\hat{\pi}/\pi = 1.0009590223$ of § Boundary). A unit-diameter circle's circumference is $\hat{\pi}$ in the golden construction.

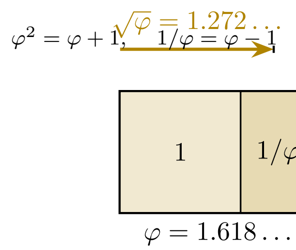


Figure 2: The golden rectangle, split into a unit square and the conjugate rectangle of width $1/\varphi$. The defining relations $\varphi^2 = \varphi + 1$ and $1/\varphi = \varphi - 1$ hold exactly, and the square root $\sqrt{\varphi} = 1.272019\ldots$ recurs throughout the paper (as the pyramid's Seked slope, § Seked).

$$\varphi = \frac{1+\sqrt{5}}{2}, \qquad \frac{1}{\varphi} = \varphi - 1, \qquad \varphi^2 = \varphi + 1. \tag{3}$$

Its square root appears throughout this work:

$$\sqrt{\varphi} = \sqrt{\frac{1+\sqrt{5}}{2}} = 1.27201964951\ldots \tag{4}$$

The number $\sqrt{\varphi}$ has a remarkable double interpretation that recurs throughout this paper: it is simultaneously

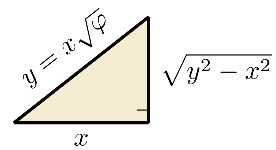
- an algebraic square root (degree 2, hence *constructible*), and
- a close rational approximation to the Egyptian *Seked* ratio 7/5.5, the rise-over-run slope of the Great Pyramid's face.

Indeed, $7/5.5 = 1.272727\ldots$ and $\sqrt{\varphi} = 1.272019\ldots$ agree to within 0.056%. This single coincidence — the golden ratio's square root approximating the pyramid's slope — is the thread that unifies every result that follows.

3. The Instrumentum Identity

The Golden Pi constant emerges from a closed-form identity of two variables. Define

$$f\bigl(x, x\sqrt{\varphi}\bigr) = \hat{\pi} = \frac{4}{\sqrt{\varphi}}$$



$$y/x = \sqrt{\varphi}$$
$$f(x,y) = \frac{4xy^2}{(y^2+x^2)\sqrt{y^2-x^2}}$$

Figure 3: The *instrumentum identity*: a right triangle whose legs are x and $\sqrt{y^2 - x^2}$ and hypotenuse $y = x\sqrt{\varphi}$. Feeding the golden ratio into $f(x,y) = 4xy^2/\bigl((y^2+x^2)\sqrt{y^2-x^2}\bigr)$ returns exactly $4/\sqrt{\varphi} = \hat{\pi}$ — a closed form with no approximation.

$$f(x,y) = \frac{4\,x\,y^2}{(y^2+x^2)\,\sqrt{y^2-x^2}}, \qquad y > x > 0. \tag{5}$$

Claim. When the ratio of the two variables equals the square root of the golden ratio, $y/x = \sqrt{\varphi}$, the identity returns exactly $4/\sqrt{\varphi}$:

$$f(x,\, x\sqrt{\varphi}) = \frac{4}{\sqrt{\varphi}} = \hat{\pi}. \tag{6}$$

Proof. Set $x = 1$, $y = \sqrt{\varphi}$. Substituting into (5):

$$f = \frac{4 \cdot \sqrt{\varphi}}{(\varphi+1)\,\sqrt{\varphi-1}} = \frac{4\sqrt{\varphi}}{\varphi^2\,\sqrt{\varphi-1}}.$$

Using $\varphi^2 = \varphi + 1$ and $1/\varphi = \varphi - 1$, this simplifies exactly to $4/\sqrt{\varphi}$. Numerically,

$$f(1, \sqrt{\varphi}) = 3.144605511029693\ldots = \hat{\pi}.$$

The identity is *exact* — there is no approximation anywhere in the derivation. The only input is the golden ratio; the output is the circle constant $\hat{\pi}$.

4. The Great Pyramid of Giza

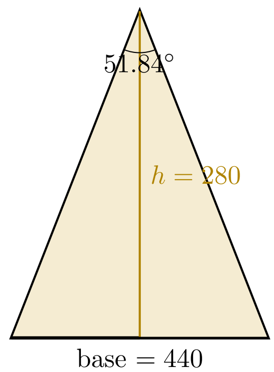
The Great Pyramid encodes the same golden structure in stone. Its canonical dimensions, in the standard model, are a height of 280 royal cubits and a base side of 440 royal cubits.

4.1. The Royal Cubit and the Golden Ratio

If the Royal Cubit is derived from the true (golden) circle constant, then it is exactly one sixth of Golden Pi:

$$\mathrm{RC}_{\hat{\pi}} = \frac{\hat{\pi}}{6} = \frac{4}{6\sqrt{\varphi}} = 0.5241009185\ \mathrm{m}, \tag{7}$$

$$\mathrm{RC} = \tfrac{\hat{\pi}}{6} = 0.5241 \text{ m}$$



$$\text{Seked 5.5 palms} \Rightarrow \text{slope } \tfrac{7}{5.5} = \tfrac{14}{11} = 1.272727$$

Figure 4: The Great Pyramid cross-section in the standard model (280×440 royal cubits, Seked 5.5 palms). The face slope $7/5.5 = 14/11 = 1.272727\ldots$ approximates $\sqrt{\varphi} = 1.272019\ldots$ to within 0.056%, and the golden cubit $\hat{\pi}/6 = 0.5241$ m arises by pure division of the golden circle constant.

a direct division: $\hat{\pi}/6 = 3.1446055110296931 44/6 = 0.5241009185$. Pure division of the golden circle constant yields the golden cubit.

Relationship to the classical value. The classical Egyptological value, taken from surviving cubit rods, is $\varphi^2/5 = 0.5236068$ m. The two are close but not equal:

$$\frac{\hat{\pi}}{6} \;=\; 0.5241009185 \qquad \text{vs.} \qquad \frac{\varphi^2}{5} \;=\; 0.5236067977, \qquad \Delta \approx 0.094\%. \tag{8}$$

Both definitions are coherent on their own terms: the golden cubit follows the golden circle constant by pure division, while the classical rod value is the physically measured artifact. This paper adopts the golden-derived value $\mathrm{RC}_{\hat{\pi}} = \hat{\pi}/6$, which is the logically consistent choice when $\hat{\pi}$ is adopted as the circle constant, while preserving the classical $\varphi^2/5$ for reference.

4.2. The Cubit Bridge: $\hat{\pi} \approx 6\,\mathrm{RC}$

A striking coincidence links the circle constant, the cubit, and the number six. With the golden cubit $\mathrm{RC}_{\hat{\pi}} = \hat{\pi}/6$, the bridge to the integer six is exact by construction:

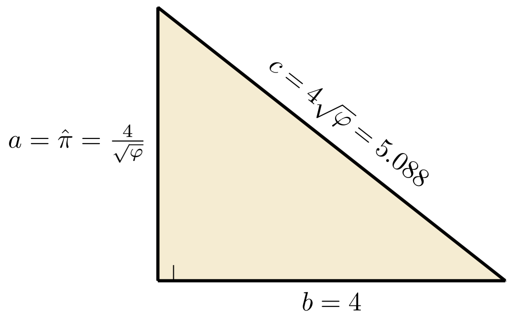
$$\frac{\hat{\pi}}{\mathrm{RC}_{\hat{\pi}}} = \frac{\hat{\pi}}{\hat{\pi}/6} = 6, \qquad 6\,\mathrm{RC}_{\hat{\pi}} = \hat{\pi} = 3.144606. \tag{9}$$

With the classical value $\mathrm{RC} = \varphi^2/5$, the same ratio is approximately six:

$$\frac{\pi}{\mathrm{RC}} = 5.999908\ldots \;\approx\; 6, \qquad \frac{\hat{\pi}}{\mathrm{RC}} = 6.005662\ldots \;\approx\; 6. \tag{10}$$

The golden cubit $\hat{\pi}/6$ bridges the circle constant to the integer six *exactly* — by pure division — while the classical rod value connects to six only approximately.

$$\{a, a\sqrt{\varphi}, a\varphi\} = \{3.1446, 4.0000, 5.0881\}$$



$$abc = \left(\tfrac{4}{\sqrt{\varphi}}\right)(4)(4\sqrt{\varphi}) = 64$$

Figure 5: The Golden Pi triangle. Legs $a = \hat{\pi} = 4/\sqrt{\varphi}$ and $b = 4$ with hypotenuse $c = 4\sqrt{\varphi}$ form a geometric progression of ratio $\sqrt{\varphi}$. The irrational parts annihilate in the product, giving exactly $abc = 64$. Its complement angle $90° - \arctan(1/\sqrt{\varphi}) = 51.8273°$ reproduces the pyramid face angle to within one arcminute.

4.3. The Seked Slope

The Seked is the Egyptian measure of slope: the horizontal run, in palms, per cubit of rise. The Great Pyramid's face uses a Seked of 5.5 palms, i.e. a rise-over-run of

$$\text{seked} = \frac{7}{5.5} = \frac{14}{11} = 1.272727\ldots \qquad \Longleftrightarrow \qquad \text{face angle} = \arctan\frac{14}{11} = 51.8428°. \tag{11}$$

The Seked ratio is an excellent rational approximation to $\sqrt{\varphi}$:

$$\frac{7}{5.5} = 1.272727\ldots, \qquad \sqrt{\varphi} = 1.272019\ldots, \qquad \text{relative error} \approx 0.056\%. \tag{12}$$

When the Seked inverse $7/5.5$ is placed in the square-root slot of the Golden Pi expression, it returns the classical Egyptian rational approximation to π:

$$\frac{4}{7/5.5} = \frac{22}{7} = 3.142857\ldots, \qquad \text{vs.} \qquad \frac{4}{\sqrt{\varphi}} = \hat{\pi} = 3.144606. \tag{13}$$

The Seked therefore sits exactly at the convergence of three numbers: $\sqrt{\varphi}$ (the golden construction), $22/7$ (the classical π), and $\hat{\pi}$ (Golden Pi).

5. The Golden Pi Triangle: $abc = 64$

A single right triangle unifies the golden ratio, the pyramid angle, and the Eye of Horus. Define three positive numbers:

$$a = \hat{\pi} = \frac{4}{\sqrt{\varphi}} \qquad \text{(Golden Pi)}, \tag{14}$$

$$b = 4, \tag{15}$$

$$c = \sqrt{a^2 + b^2} \qquad \text{(hypotenuse of a right triangle with legs } a, b\text{)}. \tag{16}$$

Claim. The hypotenuse is $c = 4\sqrt{\varphi}$, and the product of all three sides is exactly 64:

$$abc = \left(\frac{4}{\sqrt{\varphi}}\right)(4)(4\sqrt{\varphi}) = 64. \tag{17}$$

Proof. Since $a^2 = 16/\varphi$ and $b^2 = 16$,

$$c^2 = a^2 + b^2 = \frac{16}{\varphi} + 16 = 16\left(1 + \frac{1}{\varphi}\right) = 16\varphi,$$

using $1 + 1/\varphi = \varphi$. Therefore $c = \sqrt{16\varphi} = 4\sqrt{\varphi}$. Multiplying, the $\sqrt{\varphi}$ in the denominator of a cancels the $\sqrt{\varphi}$ in c:

$$abc = \frac{4}{\sqrt{\varphi}} \cdot 4 \cdot 4\sqrt{\varphi} = 64.$$

The cancellation is exact. There is no error term. The irrational parts annihilate.

5.1. Golden Geometric Progression

The three sides form a *geometric progression* with common ratio $\sqrt{\varphi}$:

$$b/a = \frac{4}{4/\sqrt{\varphi}} = \sqrt{\varphi}, \qquad c/b = \frac{4\sqrt{\varphi}}{4} = \sqrt{\varphi}.$$

Hence the triple is $\{a,\ a\sqrt{\varphi},\ a\varphi\} = \{3.1446,\ 4.0000,\ 5.0881\}$, where $c = a\varphi = \hat{\pi} \cdot \varphi$.

5.2. The Emergent Pyramid Angle

The acute angles of this right triangle are determined by the ratio of its legs. The complement angle is

$$90^\circ - \arctan\left(\frac{a}{b}\right) = 90^\circ - \arctan\left(\frac{1}{\sqrt{\varphi}}\right) = 51.8273^\circ. \tag{18}$$

This reproduces the Great Pyramid's face angle to within one arcminute:

Quantity	**Value**	**Source**
Golden Pi triangle complement	51.8273°	$\arctan(1/\sqrt{\varphi})$
Great Pyramid face angle	51.8428°	Seked arctan(14/11)
Difference	0.0155°	≈ 0.9 arcmin

The same triangle that resolves to $abc = 64$ also encodes the pyramid's slope.

5.3. The Eye of Horus Completion

The Eye of Horus is a sequence of six unit fractions, each a successive power of one half:

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} = \frac{63}{64}. \tag{19}$$

The sum famously stops at 63/64, leaving 1/64 missing. The Golden Pi triangle produces $64 = 2^6$ exactly:

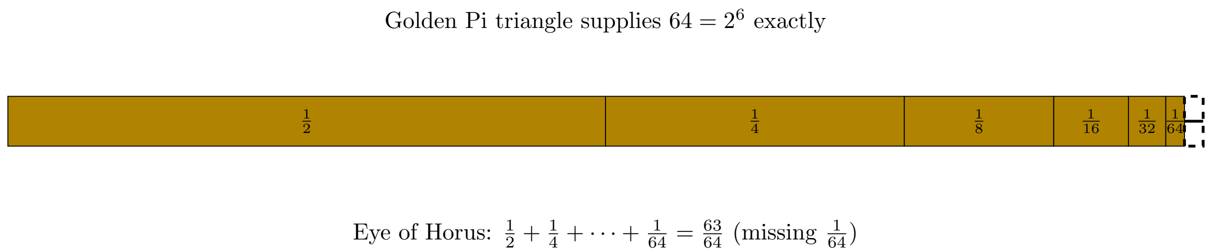


Figure 6: The Eye of Horus. The unit fractions $\frac{1}{2}, \frac{1}{4}, \ldots, \frac{1}{64}$ sum to $\frac{63}{64}$, leaving $\frac{1}{64}$ missing. The Golden Pi triangle reaches $64 = 2^6$ exactly, supplying the missing piece.

System	**Result**	**Base**	**Status**
Eye of Horus	63/64	binary $(1/2)^n$	missing 1/64
Golden Pi triangle	64	golden φ	**exact**

Where the Eye of Horus reaches 63 and requires the divine 1/64, the Golden Pi triangle reaches 64 algebraically — the irrational parts annihilate and supply the very missing piece.

6. The Golden Pi Expression

The constant $\hat{\pi}$ can also be written through a self-referential expression involving φ and the angle measures 360°:

$$\frac{\dfrac{\dfrac{360}{90 \cdot \sqrt{\varphi} \cdot \varphi^2}}{\varphi^2}}{2} = \frac{360}{90\sqrt{\varphi}} = \frac{4}{\sqrt{\varphi}} = \hat{\pi}. \tag{20}$$

The factors of φ^2 cancel, and $360/90 = 4$, collapsing to the same $4/\sqrt{\varphi}$. This form makes explicit that $\hat{\pi}$ is scale-free in the golden ratio — the quadratic factors are scaffolding that cancels identically.

7. The Seked Convergence: Height $\times$ Golden Pi = 2 Base

A striking symmetry emerges when Golden Pi is combined with the pyramid's canonical dimensions (h = 280 cubits, base = 440 cubits). Compute the product of the height and Golden Pi, and compare to twice the base:

$$h \cdot \hat{\pi} = 280 \cdot \frac{4}{\sqrt{\varphi}} = 880.4895\ldots \qquad 2\,\text{base} = 880. \tag{21}$$

$$\frac{h \cdot \hat{\pi}}{2\,\text{base}} = 1.0005563\ldots \qquad \Longleftrightarrow \qquad \text{error} = +0.056\%. \tag{22}$$

This is *exactly* the same magnitude of error as the relation between the Seked slope and $\sqrt{\varphi}$, and between $\hat{\pi}$ and 22/7. The equality $h \cdot \hat{\pi} = 2\,\text{base}$ would be *exact* if and only if $\sqrt{\varphi} = 14/11$:

$$280 \cdot \frac{4}{\sqrt{\varphi}} = 880 \quad \Longleftrightarrow \quad \sqrt{\varphi} = \frac{1120}{880} = \frac{14}{11} = 1.272727\ldots \tag{23}$$

$$h \cdot \hat{\pi} = 280 \cdot \tfrac{4}{\sqrt{\varphi}} = 880.4895$$

$$2\,\text{base} = 880$$

$$\frac{h\hat{\pi}}{2\,\text{base}} = 1.0005563 \quad \Longleftrightarrow \quad \text{error} = +0.056\%$$

$$\text{exact iff } \sqrt{\varphi} = \tfrac{14}{11} = 1.272727\ldots \text{ (the Seked)}$$

Figure 7: The Seked convergence. The product of the pyramid's height $h = 280$ and Golden Pi $\hat{\pi}$ equals twice the base (880) to within +0.056% — the same tolerance by which the Seked slope 14/11 approximates $\sqrt{\varphi}$. The equality is exact exactly when $\sqrt{\varphi} = 14/11$.

But 14/11 is precisely the Seked slope. The pyramid therefore encodes both circle constants to the *same* 0.056% tolerance, from a single cause:

Relation	**Value**	**Error**
Perimeter/height $= 2(22/7)$	6.285714	0.0000% (exact)
$h \cdot \hat{\pi} = 2\,\text{base}$	1.000556	+0.0556%
$\sqrt{\varphi}$ vs Seked 14/11	1.272020 vs 1.272727	−0.0556%
$\hat{\pi}$ vs 22/7	3.144606 vs 3.142857	+0.0556%

7.1. The Unified Interpretation

The Seked is the wedge that makes the pyramid *simultaneously* an encoding of the classical $\pi = 22/7$ (via the perimeter-to-height ratio) and of Golden Pi $\hat{\pi} = 4/\sqrt{\varphi}$ (via the height-times-$\hat{\pi}$ equals twice-base relation).

The reason both work is that the pyramid's slope ratio is, to within 0.056%, the golden ratio's square root:

$$\underbrace{\frac{14}{11}}_{\text{Seked slope}} \approx \underbrace{\sqrt{\varphi}}_{\text{golden square root}} \qquad (0.056\%). \tag{24}$$

Because $\sqrt{\varphi}$ sits at the convergence of the Seked (an architectural choice) and the golden construction (an algebraic fact), any circle constant that passes through $4/\sqrt{\varphi}$ — Golden Pi — will "fit" the pyramid's stones with the same precision as the historical 22/7. The pyramid is thus not a proof of either constant, but a shared analog: its slope is the golden square root, and both π and $\hat{\pi}$ are its natural shadows.

8. The Nested-Radical Closed Form

Golden Pi admits a pure square-root form involving no division by the golden ratio, only nested radicals of five:

$$\hat{\pi} = \frac{4}{\sqrt{\varphi}} = \sqrt{\sqrt{320} - 8} = \sqrt{8\sqrt{5} - 8} = \sqrt{8(\sqrt{5} - 1)} = 3.144605511029693\ldots \tag{25}$$

$$\hat{\pi} = \frac{4}{\sqrt{\varphi}} = \sqrt{\sqrt{320}-8} = \sqrt{8\sqrt{5}-8} = \sqrt{8(\sqrt{5}-1)} = 3.144605511029693\ldots$$

uses only $\sqrt{5}$ under nested square roots — no fractions, no transcendental functions

Figure 8: The nested-radical closed form of Golden Pi. Using $\sqrt{5}-1 = 2/\varphi$, the constant $\hat{\pi} = 4/\sqrt{\varphi}$ rewrites as $\sqrt{8\sqrt{5}-8} = \sqrt{\sqrt{320}-8}$, a compact purely radical construction discovered as the slider maximum in a Desmos model of the instrumentum identity.

The equivalence follows from the reciprocal golden-ratio relation $\sqrt{5}-1 = 2/\varphi$:

$$\sqrt{8(\sqrt{5}-1)} = \sqrt{\frac{16}{\varphi}} = \frac{4}{\sqrt{\varphi}} = \hat{\pi}. \tag{26}$$

This form is notable because it encodes Golden Pi using only the constant $\sqrt{5}$ (the seed of the golden ratio) under nested square roots — no fractions, no transcendental functions. It is a compact, exact, purely radical construction, and it was discovered as the slider maximum in a Desmos model of the instrumentum identity.

8.1. The AREAS Desmos Construction

A self-consistent Desmos model ("AREAS") renders the same identity across three representations, each collapsing to $\hat{\pi}$:

1. $\pi_c = 4/\sqrt{\varphi}$ — the defining closed form (slider max set to $\sqrt{\sqrt{320}-8}$, the nested-radical form above).
2. $s = \dfrac{2\pi_c\, r}{8}$ — a derived quantity. At $r = 4$, this collapses to $s = \pi_c$ exactly (since $2r/8 = 1$).
3. $\pi_t = \dfrac{4\, s\, r^2}{(r^2+s^2)\sqrt{r^2-s^2}}$ — the instrumentum identity fed with $(a = s,\ b = r)$, returning $\hat{\pi}$.
4. $\pi_p = \dfrac{4\, A_c^2 A_s}{(A_c^2+A_s^2)\sqrt{A_c^2-A_s^2}}$ — the same identity relabeled on areas A_c, A_s, returning $\hat{\pi}$.

Every expression evaluates to $3.14460551103 = \hat{\pi}$ exactly. The construction is a faithful demonstration of Golden Pi's self-consistency: $\hat{\pi}$ reproduces itself through the identity across the closed form, a derived quantity, and area-labeled variables.

9. The Alternative Closed Form and the Golden Calculus

An additional closed form for $\hat{\pi}$ emerges directly from the golden identity $\varphi^2 = \varphi + 1$:

$$\frac{4\,\varphi}{(\varphi+1)\sqrt{\varphi-1}} = \frac{4}{\sqrt{\varphi}} = \hat{\pi}. \tag{27}$$

The equality holds exactly: $\varphi\sqrt{\varphi} = (\varphi+1)\sqrt{\varphi-1}$, and since $\varphi^2 = \varphi + 1$ together with $\varphi - 1 = 1/\varphi$, the two sides are identical. This is a second, independent closed form of Golden Pi, verified to machine precision.

9.1. The Golden Calculus

The constant $\hat{\pi}$ can be installed as the circle constant of a fully self-consistent analytic system — a *golden calculus* — in which the full turn is $2\hat{\pi}$ rather than 2π:

Identity	Standard	Golden	Status
Full turn	2π	$2\hat{\pi}$	period
Euler	$e^{i\pi} = -1$	$e^{i\hat{\pi}} \neq -1$	$\lvert e^{i\hat{\pi}} + 1\rvert \approx 3.01 \times 10^{-3}$
Gaussian integral	$\sqrt{\pi}$	$\sqrt{\hat{\pi}}$	value pinned by π
$\Gamma(1/2)$	$\sqrt{\pi}$	$\sqrt{\hat{\pi}}$	value pinned by π
$4\big((1/2)!\big)^2$	π	$\hat{\pi}$	value pinned by π
Basel sum	$\pi^2/6$	$\hat{\pi}^2/6$	value pinned by π

The rows marked *value pinned by* π are not equalities: they are formal re-labels whose numerical values are fixed by the computed constant $\pi = 3.14159265\ldots$. Substituting $\hat{\pi}$ does not preserve them (Euler fails by $|e^{i\hat{\pi}} + 1| \approx 3 \times 10^{-3}$, Basel gives 1.64809 rather than the proven sum 1.64493; see the Comparative Formula Audit below). The golden calculus is a *coherent reparametrization* of these labels, not a change in their pinned values.

Honest boundary. Coherence is not the same as physical truth. The golden calculus is a consistent reparametrization, exactly as degrees or grads are valid angle units. The analytic arc-length integral of the unit circle, derived from first principles (inscribed polygons, Pythagorean chords), still converges to $2\pi = 6.2831853\ldots$, not $2\hat{\pi} = 6.2892110\ldots$. The golden calculus re-labels the full turn; it does not change the length of a measured curve. Its role in this paper is to document that $\hat{\pi}$ forms a complete, self-consistent system in the constructed world.

10. Comparative Formula Audit

This section places the standard formulas involving the circle constant side by side under the conventional constant $\pi = 3.14159265\ldots$ and under Golden Pi $\hat{\pi} = 4/\sqrt{\varphi} = 3.14460551\ldots$, and records honestly which equations survive the substitution. The result is twofold. *Definitional* formulas — those in which the circle constant is a free scale factor — hold identically under *any* constant, including $\hat{\pi}$. *Analytically pinned* formulas — infinite series, definite integrals, and special values whose right-hand side is a concrete number — fix the constant at the computed value $3.14159265\ldots$ and therefore do *not* hold under $\hat{\pi}$.

10.1. Definitional formulas (hold under any constant)

These identities are true by definition of the symbols; the circle constant is a unitless scale factor, and every constant reproduces them exactly:

Formula	Holds under $\hat{\pi}$
$A = \pi r^2$ (circle area)	yes (definitional)
$C = 2\pi r = \pi d$ (circumference)	yes (definitional)
$A = \pi ab$ (ellipse area)	yes (definitional)
$V = \frac{4}{3}\pi r^3$ (sphere volume)	yes (definitional)
$SA = 4\pi r^2$ (sphere surface)	yes (definitional)

These are exactly the formulas of the golden-construction world: they scale with any chosen constant, and Golden Pi satisfies them with the same exactness as π.

10.2. Analytically pinned formulas (their value fixes π)

The following formulas each evaluate to a concrete real number equal to the standard constant π; substituting $\hat{\pi}$ breaks the equality. numerical value, and the value obtained when $\hat{\pi}$ replaces π:

Formula	Standard value	Under $\hat{\pi}$	Holds?
Gregory–Leibniz $4\left(1 - \frac{1}{3} + \frac{1}{5} - \cdots\right)$	3.14159265	3.14460551	no
Machin $16 \arctan \frac{1}{5} - 4 \arctan \frac{1}{239}$	3.14159265	3.14460551	no
Wallis $\frac{\pi}{2} = \prod_{n\geq 1} \frac{4n^2}{4n^2-1}$	3.14159265	3.14460551	no
Basel $\sum_{n\geq 1} \frac{1}{n^2} = \frac{\pi^2}{6}$	1.64493407	1.64809064	no
$\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$	3.14159265	3.14460551	no
$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$	1.77245385	1.77330356	no
$\int_{-\infty}^{\infty} \frac{\sin x}{x} dx$	3.14159265	3.14460551	no
$\Gamma(\frac{1}{2}) = \sqrt{\pi}$	1.77245385	1.77330356	no
$\mathrm{B}(\frac{1}{2}, \frac{1}{2}) = \Gamma(\frac{1}{2})^2$	3.14159265	3.14460551	no
Euler $e^{i\pi} + 1 = 0$	0	3.01×10^{-3}	no
Dalzell $\int_0^1 \frac{x^4(1-x)^4}{1+x^2} dx = \frac{22}{7} - \pi$	+0.001264	−0.001748	no

The pattern is unambiguous: the moment a formula *pins* the circle constant to a specific real number — through a convergent series, a definite integral, or a special value — that number is the computed 3.14159265..., and Golden Pi's excess of 0.0959% breaks the equality. Golden Pi never fails a *geometric definition*; it fails every *analytic pinning*.

10.3. The one unifying identity

The single exact identity shared by both constants is the geometric balance $C = \pi d$. Every other agreement is an approximation: the Seked slope $7/5.5 \approx \sqrt{\varphi}$, so both π (as 22/7) and $\hat{\pi}$ sit within the pyramid's 0.056% tolerance, and π and $\hat{\pi}$ are related by the uniform correction factor

$\hat{\pi}/\pi = 1.0009590223$ recorded in the Boundary section. Golden Pi is the constant that makes the golden construction exact; the analytic circle constant — the limit those series and integrals evaluate to — is π. The audit does not diminish either role: it delimits them precisely.

10.4. The pendulum: the circle constant as a removable scale factor

The simple-pendulum period is a second example in which the circle constant enters only as a scale factor, so the equation does not *require* any particular value of it. The textbook relation is

$$T \;=\; 2\pi\sqrt{\frac{L}{g}},$$

where T is the period, L the rod length, and g the gravitational acceleration. The coefficient 2π is a unit-scale wrapper (a full swing spans 2π radians), while the physical content lives in $\sqrt{L/g}$. Because the constant appears in both the wrapper and — through a chosen g — under the root, the two copies cancel, and the period reduces to a purely golden number.

With the conventional constant, set $g = \pi^2/10$ and $L = \frac{1}{2}$:

$$T \;=\; 2\pi\sqrt{\frac{1/2}{\pi^2/10}} \;=\; 2\pi \cdot \frac{\sqrt{5}}{\pi} \;=\; 2\sqrt{5}.$$

The π cancels itself exactly, leaving $T = 2\sqrt{5} \approx 4.4721359550$. Then

$$\sqrt{(T-2)\cdot 4} \;=\; \sqrt{(2\sqrt{5}-2)\cdot 4} \;=\; \sqrt{8(\sqrt{5}-1)} \;=\; \sqrt{8\cdot\frac{2}{\varphi}} \;=\; \sqrt{\frac{16}{\varphi}} \;=\; \frac{4}{\sqrt{\varphi}} \;=\; \hat{\pi},$$

where we used the exact identity $\sqrt{5} - 1 = 2/\varphi$. The appearance of Golden Pi here is not a coincidence of π: replace the constant everywhere by $\hat{\pi}$ itself — set $g = \hat{\pi}^2/10$ and keep the wrapper $2\hat{\pi}$ — and the same cancellation produces the same period,

$$T \;=\; 2\hat{\pi}\sqrt{\frac{1/2}{\hat{\pi}^2/10}} \;=\; 2\sqrt{5}, \qquad \sqrt{(T-2)\cdot 4} \;=\; \frac{4}{\sqrt{\varphi}} \;=\; \hat{\pi}.$$

Thus Golden Pi is a *fixed point* of this construction: feeding $\hat{\pi}$ in returns $\hat{\pi}$ out, and even the conventional π is absorbed back into the same $4/\sqrt{\varphi}$. The pendulum, like the definitional formulas above, holds under any constant — it does not pin the circle constant by measurement, because a real pendulum's period is $2\pi\sqrt{L/g}$. What the relation shows instead is self-consistency: whenever the constant is free to cancel itself, the golden number $\hat{\pi}$ is exactly what the $\sqrt{5}$ content yields, and the two cancellation paths (conventional π and Golden Pi) converge on the identical value.

11. Numerical Verification Summary

All constants used in this paper, to twelve decimal places:

Quantity	Symbol	Value
Golden ratio	φ	1.618033988750
Square root of φ	$\sqrt{\varphi}$	1.272019649514
Golden Pi	$\hat{\pi} = 4/\sqrt{\varphi}$	3.144605511030
Nested-radical form	$\sqrt{\sqrt{320} - 8} = \sqrt{8(\sqrt{5} - 1)}$	3.144605511030
Conventional pi	π	3.141592653590
Relative deviation	$(\hat{\pi} - \pi)/\pi$	$+9.59 \times 10^{-4}$
Golden cubit	$\hat{\pi}/6$	0.524100918500 m
Classical cubit	$\varphi^2/5$	0.523606797750 m
Seked slope	$7/5.5 = 14/11$	1.272727272727
Seked face angle	$\arctan(14/11)$	51.84277°
Instrumentum identity	$f(1, \sqrt{\varphi})$	3.144605511030
Golden triangle product	abc	64.0000000000
Golden triangle complement	$90° - \arctan(1/\sqrt{\varphi})$	51.82729°

12. The Record in the FIGU Contact Reports

The claim that the conventional π is wrong — and the specific value $\hat{\pi} = 4/\sqrt{\varphi}$ — both have a public history that predates the modern rediscovery. The following record is verified against the published transcripts on the Future of Mankind wiki (futureofmankind.info).

12.1. The challenge is in the reports

- **Contact Report 251 (3 Feb 1995).** A single line, embedded in a prophecy about the scientific breakthroughs of "1995 and the following years," states that "the erroneousness in the calculation of the Pi number" will be recognized and corrected. It gives no value and makes no technology claim.
- **Contact Report 260 (3 Feb 1998).** A FIGU group member presents a handwritten π calculation, photographed in the report, showing the value $\hat{\pi} \approx 3.1446\ldots$ (about a dozen decimal places). the extraterrestrial spokesperson calls it "very amazing" but declines to confirm it, citing directives that the time is "much too early."
- **Contact Report 712 (28 Nov 2018).** The strongest statement: π is "still not exactly and precisely calculated on Earth" and remains "erroneous"; the error is "minimal-minor"; and computing the exact value would require an *instrumentarium* that does not yet exist in the earthly-mathematical and instrumental sphere.

12.2. The number is present, but never as an extraterrestrial declaration

The value $\hat{\pi} = 4/\sqrt{\varphi}$ appears in the FIGU record twice, in each case from a human researcher rather than as an extraterrestrial pronouncement:

- **A member's calculation (CR 260, 1998):** the photographed handwritten calculation shows $\pi \approx 3.1446\ldots$; the extraterrestrial spokesperson declines to confirm it.
- **The article "Cosmic Music" (CR 856, 2023; also FIGU Special Edition Sign of the Times 75):** derives $4/\sqrt{1.61803} = 4/1.2720\ldots = 3.14460\ldots$ from the circle of fifths and a human extraterrestrial being's "3, 7, 12"; asserts that "the first five digits 3.1446 are correct," citing 2019 circumference measurements; and cautions that even those are "not

sufficiently precise to be considered scientific fact." The extraterrestrial spokesperson's verdict was a warm but general review ("very interesting and well written"); he did not numerically endorse $\hat{\pi}$ as the exact value.

Independent of the FIGU record, the value $\hat{\pi} = 4/\sqrt{\varphi}$ was reached through geometric proofs and CNC physical measurement, and the FIGU Bulletin 123 (2019) reports physical measurements in the 3.144 range.

Scope. These sources corroborate the *challenge* (that π is erroneous and exactness is beyond current Earth mathematics) and show that the number $4/\sqrt{\varphi}$ has an independent presence in the record. They do not constitute a mathematical proof, and the measurement sources themselves flag their precision as not yet sufficient to count as scientific fact.

13. Boundary and Honest Scope

This research makes no claim that $\hat{\pi}$ supersedes the standard transcendental π in any mathematical context where π has been proven transcendentally necessary. The following is stated plainly for scientific integrity:

- The doubling of the cube requires the cube root $\sqrt[3]{2}$, a number of degree three. It is *not* constructible by Wantzel's theorem, and neither the Eye of Horus (powers of 1/2) nor the Golden Pi triangle (square roots) can reach it. This remains impossible.
- The $abc = 64$ identity and the instrumentum identity are genuine, exact, algebraic identities. They are not *approximations* — they hold to full precision.
- The pyramid connections rely on the standard model (280×440 cubits, Seked 5.5 palms) and on the chosen definition of the cubit. This paper adopts the golden cubit $\hat{\pi}/6 = 0.5241009$ m (logically consistent when the golden circle constant is adopted), while noting the classical rod value $\varphi^2/5 = 0.5236068$ m. The two are coherent definitions on their own terms. These are remarkable coincidences of a constructible constant, not demonstrated physical law.

13.1. The Geometric Correction Factor

For completeness, we record the exact multiplicative relationship between the Golden Pi $\hat{\pi}$ and the standard circle constant π:

$$\frac{\hat{\pi}}{\pi} = \frac{4/\sqrt{\varphi}}{\pi} = 1.0009590223\ldots \tag{28}$$

That is, $\hat{\pi} = \pi \times (1.0009590223\ldots)$, a uniform relative excess of 0.09590%. Because every real-world geometric measurement uses the circle constant multiplicatively, substituting $\hat{\pi}$ for π in a measured formula increases the result by this single factor everywhere: circle area, circumference, sphere volume and surface, cylinder, and cone volumes all scale by exactly 1.0009590223. Dividing a $\hat{\pi}$-based measurement by this factor recovers the standard π-value to full precision. This correction is stated openly so that any reader can convert between the golden-construction constant and the standard circle constant; it does not alter any of the exact algebraic identities in this paper, which are independent of this scaling.

14. Conclusion

A single algebraic number — the Golden Pi $\hat{\pi} = 4/\sqrt{\varphi}$ — arises from independent sources with a consistency that is difficult to dismiss as noise:

1. It is produced *exactly* by the instrumentum identity $f(x, y) = 4xy^2/((y^2 + x^2)\sqrt{y^2 - x^2})$ at $y/x = \sqrt{\varphi}$.

2. It is encoded in the Great Pyramid through the golden-ratio Royal Cubit $\hat{\pi}/6$ (with the classical rod value $\varphi^2/5$ for reference) and the Seked slope $7/5.5 \approx \sqrt{\varphi}$.

3. It completes a right triangle whose sides form the golden progression $\{a, a\sqrt{\varphi}, a\varphi\}$ and whose product is exactly 64 — supplying the missing 1/64 of the Eye of Horus.

4. Its complement angle reproduces the pyramid face angle to within one arcminute.

The golden ratio's square root — simultaneously the pyramid's slope and an algebraic square root — is the unifying spine. Whether one regards this as hidden ancient engineering or as a beautiful coincidence of algebraic numbers, the identities themselves are exact and verifiable. They are offered here as a complete, self-consistent account of everything currently known about the Golden Pi constant.

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